Doing Things With Derivatives

Math 130 - Essentials of Calculus

10 March 2021

Now You Try It!

Example

A manufacturer of power supplies estimates that it will incur a total cost of $C(q) = 2500 + 4q + 0.005q^2$ when producing q power supplies, and it will collect $R(q) = 16q - 0.002q^2$ dollars in revenue.

1. *Write a function for the profit P the manufacturer can expect after producing q power supplies.*
2. *Find the marginal cost and marginal revenue functions.*
3. *How many power supplies should the manufacturer produce in order to maximize profit.*

Demand Curves

There is normally a relationship between the price of a product or service and the number of units that can be sold. Let $p = D(q)$ be the price per unit that a company can charge if it sells q units. This function D is called the **demand function** (also called a *price function*) and its graph is called the **demand curve.**

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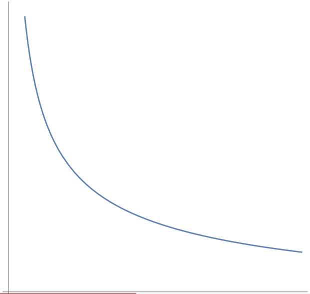

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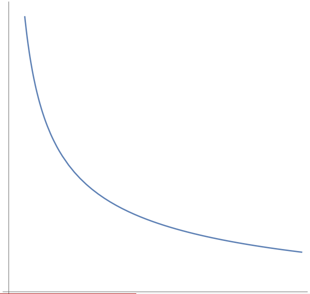

Because revenue is the number of units sold times the price per unit, the revenue can be found as

$$R(q) = q \cdot D(q).$$

Maximizing Profit

Example

A company has cost and demand functions

$$C(q) = 84 + 1.26q - 0.01q^2 + 0.00007q^3 \quad \text{and} \quad D(q) = 3.5 - 0.01q.$$

1. *If the price of each unit is $1.20, how many units will be sold?*

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1. *If the price of each unit is* $1.20, *how many units will be sold?*
2. *Determine the production level that will maximize profit for the company.*

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$$C(q) = 680 + 4q + 0.01q^2 \quad \text{and} \quad p = 12 - \frac{q}{500}.$$

Find the production level that will maximize profit.

The Quotient Rule

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Theorem (The Quotient Rule)

If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$

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4. $L(t) = \dfrac{t^2}{3t^2-2t+1}$

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